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ABSTRACT

The purposes of this paper are to present research related to the development of new instructional games (named Survivor's Games), and measure the effectiveness of these games to help students (K-5) master basic fact operations (single-digit additions or factor and single- or double digit sums or products), stimulate the exploration of mathematical ideas, and use algebraic thinking at different levels of understanding. Results show that playing the game had a positive effect on students' mastery of the basic addition facts, and the relationship between mastery of basic facts and performance development of algebraic thinking. (Author/KHR)

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Research Findings from Games Involving Basic Fact Operations and Algebraic Thinking at a PDS

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Research Findings from Games Involving Basic Fact Operations and Algebraic Thinking at a PDS

The belief that mathematics needs to be meaningful and the idea that children construct their own mathematical knowledge do not rule out the students' need for practice of procedures, like the need to memorize basic facts (Cathcart, Pothier, Vance, and Bezuk 2001). Practice contributes significantly to making routine procedures automatic. This results in a more efficient and accurate execution of a procedure, and the expenditure of less mental effort (Hiebert 1990). However, practice does not have to be dull or boring. Games are self-motivating and fun, and very effective and useful for providing practice.

Another important task of teaching mathematics is to help students develop an understanding of algebraic thinking. A fundamental aspect of this task is students' knowledge of the concept of variable. The concept of variable is considered to be one of the few central themes, which serve to organize, unify and give coherence to mathematics at all levels (Cooney, Davis, & Henderson, 1975). This is more evident when this concept is included as one of the most important concepts in the Principle and standards for school mathematics (NCTM, 2000), and when the abundant evidence of students' misconceptions of this concept is considered (Booth, 1984, 1988; Carpenter, et. al, 1981; Clement, Lockhead, & Monk, 1981; Hart, Brown Kerslake, Kücheman, & Ruddock, 1985; Herscovics, & Kieren, 1980; Kücheman, 1978; Ortiz, 1988; Ortiz, & MacGregor, 1990; Stacey, & MacGregor, 1997; Swafford, & Langrall, 2000; Tonnesen, 1980; Usiskin, 1988, 1997; Wagner, 1981).

The purposes of this paper are to present research related to the development of new instructional games (named Survivor's Games), and measure the effectiveness of these games to help students (Kindergarten to fifth grade) master basic fact operations (single-digit addends or factor and single- or double digit sums or products; for example $3 + 5 = 8$, $8 - 3 = 5$, $4 \bullet 3 = 12$, or $12 \div 3 = 4$), stimulate the exploration of mathematical ideas, and use algebraic thinking (utilizing variables) at different levels of understanding.

Methods

Participants

The participants of the study were students from Kindergarten to fifth grade from an urban public school (Sanford, Florida) during the last two months of the spring 2002 semester. This school is located in a predominately lower to middle class neighborhood. The students in this school are randomly assigned to their respective groups at the beginning of the semester. The study included six groups of students from teachers who volunteered to participate: Kindergarten (one group with 16, 8 males and 8 females, and another group with 18 students, 9 males and 9 females), first grade (one group with 24 students, 9 males and 15 females), second grade (one group 19 students, 10 males and 9 females), third grade (one group with 24 students, 16 males and 8 females), fourth grade (one group with 21 students, 9 males and 12 females), and fifth grade (one group with 23 students, 8 males and 15 females). This gave a total of 145 students.

Instruments

Pre-tests and post-tests were administered to each of the students participating in this study. The regular classroom teachers selected the levels of the pre- and post-tests. One pre-test involved basic facts operations and another pre-test involved algebraic thinking (see Appendix). Each pre-test was administered to each group in the following manner:

- **Kindergarten:** Pre-test Addition Basic Facts for Sums Up to 5: Level I (21 items)
  Pre-test Algebraic Thinking Involving Addition Level I (10 items)
- **First Grade:** Pre-test Addition Basic Facts for Sums Up to 10: Level II (64 items)
  Pre-test Algebraic Thinking Involving Addition Level II (18 items)
- **Second Grade:** Pre-test Addition Basic Facts for Sums Up to 18: Level III (100 items, included all possible addition basic fact combinations)
  Pre-test Algebraic Thinking Addition Level II (18 items)
- **Third Grade:** Pre-test Multiplication Basic Facts for Products Up to 81: Level IV (100 items, included all possible multiplication basic fact combinations)
  Pre-test Algebraic Thinking Involving Multiplication Level III (18 items)
- **Fourth Grade:** Pre-test Multiplication Basic Facts for Products Up to 81: Level IV (100 items, included all possible basic multiplication fact combinations)
  Pre-test Algebraic Thinking Multiplication Level III (18 items)
- **Fifth Grade:** Pre-test Multiplication and Division Basic Facts for Products and Quotients Up to 81: Level IV (190 items, included all possible multiplication and division basic fact combinations)
  Pre-test Algebraic Thinking Involving Multiplication and Division Level III (34 items)

The students were allowed to use manipulatives to solve the different problems and enough time to finish all the items. The emphasis was on understanding rather than on speed. In the pre-tests involving basic facts, the students were asked to find the sums, products or quotients

of several exercises. In the pre-tests involving algebraic thinking, the students were asked to find the number for a variable that will solve an equation. These problems may include one, two, or three variables. The variables were represented by symbols (♥ = heart, ♣ = clover, or ♦ = diamond) or letters (X, Y, or Z). Since students in Kindergarten, first-, and second-grades are being introduced to the use of letters for reading and writing, symbols were used instead of letters to represent variables. These letters and symbols where used to represent possible values which were elements of the set of whole number: {0, 1, 2, 3, . . . n}. The purpose of the algebraic thinking tests was to analyze students' knowledge of the use of variables involved in the games presented in this study. The items for the algebraic thinking pre-tests represented situations the students could encountered during the treatment while playing the game. The students were expected to transfer what they learned during the game into the test situation. These same pre-tests were administered as post-tests at the end of the treatment.

Procedures

Both pre-tests were administered to all students the week before starting the treatment. The following week the students participated in different levels of the game as selected by their teachers for five days:

- **Kindergarten:** Survivor's Game Level I (For Sums Up to 5)
- **First Grade:** Survivor's Game Level II (For Sums Up to 10)
- **Second Grade:** Survivor's Game Level III (For Sums Up to 18)
- **Third Grade:** Survivor's Game 1 Level IV (For Products Up to 81)
- **Fourth Grade:** Survivor's Game 1 Level IV (For Products Up to 81)
- **Fifth Grade:** Survivor's Game 1 Level IV (For Products Up to 81), and Survivor's Game 2 Level IV (For Quotients from 0 to 81)

During these five days (not necessarily consecutive days in a one- to two-week expand) the students played the game from 15 to 20 minutes per day for Kindergarten and first-grade, and from 30 to 45 minutes per day for second- to fifth-grade. The students were randomly assigned to the different small groups before playing the treatment started. The first day of the study, the game was presented by the research to each group of students. Kindergarten and first-grade students were involved in learning centers of three students each and were closely supervised by the classroom teacher or the researcher as the students played the games. The groups took turns to go through the centers. They usually played three rounds of the game per day. Second- to fifth-grade students played the games in groups of three or four students each day. The whole class played at the same time in their respective groups. These students usually played four to six rounds of the game. After this, the post-test was administered to each student.

The data collected was decoded from the pretests and post-tests in Statistical Package for the Social Sciences (SPSS) file and analyzed using SPSS version 10.0. Paired-Sample t-test procedures were used to analyze the possibility of significant differences between the students' performance in the pre-tests and post-tests by grade level. Pearson Correlation Coefficients were calculated to analyze the strength of the relationship between performance in the pre-tests for basic facts and performance in the post-tests for algebraic thinking. Also, the research kept a journal including observations and annotations of the students' performance as they played the games. This data was also analyzed for possible patterns.

Rules and Procedures for Survivor's Game Level I: Involving Sums Up to Five

The **object of the game** was to from a correct addition sentence with three of the four dice on the game board (see fig. 1). Each group needed a game board and a set of dice to play the game. The game began by deciding who would start the game, and then the players took turns rolling the four dice (see fig. 2) once going to their left. If needed, the players used counters and/or paper and pencil to find the answer to the addition sentence (or equation) (two one-digit addends and a one- or two-digit sum) created.

Players had to use only one die per square on the game board. This left an extra die that they could not use. The game ended after each player had a turn to play. The person(s) forming a correct mathematical sentence was (were) the **SURVIVOR (s)!** After the first player plays his or her turn, the second player rolled the dice again and formed an addition sentence with the new numbers and values for the symbols on the game board. The second player did not have to use the same values for the symbols as the first player. The values changed for each roll. The same applied for each of the next players' turn.

The possible outcomes were combinations of 0, 1, 2, 3, 4, 5, ❤ (heart), ♣ (clover), ♦ (diamond), and ☹ (sad face). If ❤, ♣, or ♦ was rolled, the player needed to select a number value to represent the symbol and form a correct addition sentence with the dice. Each symbol could be used to represent the value of one of the addends, or the sum. Once a value was selected for one of the symbols, it could not be changed until the next turn. The ☹ could not be used for any sentence when rolled during the game. Two or more ☹'s during a turn would not

allow the student to form a mathematical sentence during a turn. Each of the symbols could have different number values (for example, ♥ = 4, ♣ = 2, and ♦ = 8), or the same number values (for example, (♥ = 4, ♣ = 4, and ♦ = 4). Some of the possible winning combinations are the following: 2 + 1 = 3, ♥ + 1 = 5, where ♥ = 4, ♦ + ♦ = 4, where ♦ = 2. If there is a tie, both players were declared Survivors and winners of the game. The players played the game as many times as they could with the amount of time they had. They needed to follow the same instructions presented above from the beginning for each game.

Rules and Procedures for Survivor's Game Level II: Involving Sums Up to 10

Survivor's Game Level II involved sums up to 10, a game board (see fig. 3), and seven dice (see fig. 4). The rules were the same as the ones for Survivor's Game 1. In this game, the possible outcomes were 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ♥, ♣, ♦, and ☹. The three major differences were the following: the addition of one more of each of the symbols (now we have two of each of the following: ♥, ♣, and ♦) which provided more possibilities with the number values, the sums could go up to 10 and, in some cases, higher than 10 by using the variables (10 + 3 = ♦, where ♦ = 13), player could now form one or two addition sentences. The player who could form the most addition sentences won the game. In the event of a tie, after three rounds, all of these players were declared winners for that game.

Rules and Procedures for Survivor's Game Level III: Involving Sums Up to 18

Survivor's Game Level III involved sums of up to 18, a game board (see fig, 5) and nine dice (see fig. 6). The rules for this version of the game were a bit different and provided for more challenges. The students still took turns rolling the dice and formed one two or three addition sentences on the game board, using as many dice as possible. During her turn, the player could choose to roll some or all of the dice again for two more times; however, only dice that were rolled without the ☹ can be rolled again or could be used to form the addition sentences. The player placed the die or dice with ☹ in the rectangle at the bottom of the game board (see fig, 5).

The possible outcomes were 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, ☹, ♥, ♣, and ♦. In this game, when used correctly, the numbers rolled on the dice were worth one point each, and the symbols rolled on the dice were worth two points each. The possible numbers of points were 0, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, or 15. For example, 3 + 1 = 4 was worth three points, ♥ + 3 = 12 was worth 5 points, and 3 + 4 = 5 + 2 was worth 4 points. The player with the most points after three rounds of the game was named the Survivor (see figure 7 for recording form). As for the previous game, in the event of a tie, after the three rounds, all of these players were declared winners for that game.

Rules and Procedures for Survivor's Game 1 Level IV: Involving Products Up to 81

The Survivor's Game 1 Level IV (see figure 8) used the same rules as the ones presented for the Survivor's Game Level III. However, different dice were used for this game (see figure

9). This game involved products up to 81, and, depending on the outcome for the dice, one, two or three multiplication sentences. The possible outcomes were 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 49, 54, 56, 63, 64, 72, 81, X, Y, Z, and ⊗. The number used for the dice were strategically placed to avoid products larger than the ones related to the multiplication basic facts (not larger than 9 x 9 = 81). Because of the spaces available on the dice, some products are still possible: 10 • 2 = 20, 10 • 3 = 30, 10 • 4 = 40, 12 • 2 = 24, 12 • 3 = 36, 12 • 4 = 48, 12 • 6 = 72, 16 • 3 = 48, and 2 • 27 = 54.

The player with the most points after three rounds of the game was named the Survivor (see figure 7 for recording form). As for the previous game, in the event of a tie, all of these players were declared winners for that game.

Rules and Procedures for Survivor's Game 2 Level IV: Involving Quotients From 0 to 81

The use of the Survivor's Game 2 Level IV for division basic facts was very simple. The only thing that was done was to change the multiplication signs to division signs on the game boards (see fig. 10). The same dice and rules that were used for the multiplication version were also used for the division version.

Results

Statistical Analyses

The descriptive statistics for pre-tests and post-tests for both basic facts and algebraic thinking by grade level are included in table 1. This includes mean, sample size, standard deviation, and standard error.

Table 2 presents the finding for each Paired Sample T-test involving pre-tests and post-tests by grade level. Twelve pairs are included (two for each grade level). Three of the Paired Sample T-tests were found to be significant: Pre-test for Addition Basic Facts and the Post-test for Addition Basic Facts for Kindergarten and first-grade, and Pre-test for Algebraic Thinking and the Post-test for Algebraic Thinking for first-grade.

Table 3 presents the Pearson Correlation Coefficient for each of the following relationships by grade level. The following coefficients were significant.

- All of the Pre-tests for Basic Facts and Post-tests for Basic Facts at the 0.01 level,
- Post-test for Basic Facts and Post-test for Algebraic Thinking for third-, fourth-, and fifth-grade at the 0.01 level, and
- Post-test for Algebraic Thinking and Post-test for Algebraic Thinking for first- (at the 0.05 level), second-, third-, and fifth-grade the last three at the 0.01 level).

Students' Responses for Survivor's Game Level I

The following represent some of the addition sentences made by the students during the treatment for Survivor's Game Level I:

- For 1, 4, 5, and 2 in one roll, the player formed 1 + 4 = 5, and will not use 2. In some cases the player tried to form an incorrect addition sentence like 1 + 2 = 4 without thinking if it was correct or not, the teacher helped the student clarify and correct the error.
- For 1, 2, 3, and ♣ in one roll, the player formed the addition sentence 1 + 3 = ♣, where ♣ = 4 and 2 were not used. Notice that 1 + 2 = 3 could also be formed. This was the most common use of the symbols to represent numbers in this level of the game.
- For ♥, 5, 1 and 3 in one roll, the player formed the addition sentence ♥ + 3 = 5, where ♥ = 2, and 1 were not used. This move involved the use of a missing addend which was especially difficult for, and seldom use as a possible answer by the students at this level.

- For ♥, ♣, 2, and 9 in one roll, the player formed the addition sentence ♥ + ♣ = 9, where ♥ = 3 and ♣ = 6, and 2 was not used. This one is worth five points.
- For ♥, ♣, ♦ and 5 in one roll, the player formed the addition sentence ♥ + ♣ = ♦, where ♥ = 2, ♣ = 1 and ♦ = 3 and 5 was not used. This one was worth six points.
- For 1, 1, 4, and 3 in one roll, the player could not form a correct addition sentence with these numbers, and earn points for this turn.

Students' Reponses for Survivor's Game Level II

The following represent some of the addition sentences made by the students during the treatment for Survivor's Game Level II:

- For 0, 1, 2, 4, 7, 5, and 10 in one roll, a player formed only one addition sentence, 1 + 4 = 5. This player was not able to use 0, 2, 7, and 10. With the same roll (0, 1, 2, 4, 7, 5, and 10), the player had the option of forming a different addition sentence: 5 + 2 = 7. The player still was not able to use some of the other numbers: 0, 1, 4, and 10.
- For 1, 2, 2, 4, 5, 5, and 7 in one roll, the player formed two addition sentences: 1 + 4 = 5, and 5 + 2 = 7. The player was not able to use one of the 2's.
- For 1, 2, 4, 5, 5, 7, and ♣ in one roll, the player formed two addition sentences: 1 + 4 = 5, and 5 + 2 = ♣, where ♣ = 7. The player was not able to use one of the 2's. Notice that the player could play this outcome differently: 1 + 4 = 5, and 5 + 2 = 7. In this case, ♣ was not used.
- For 0, 2, 1, 2, 3, 7, and ♣ in one roll, the player formed two addition sentences: 1 + 3 = ♣ (where ♣ = 4) and 2 + 0 = 2. The player was not able to use 7 as a possible value.
- For ♥, ♣, 5, 1, 1, 3, and 3 in one roll, the player formed two addition sentences: ♥ + 3 = 5 (where ♥ = 2), and 3 + 1 = ♣ (where ♣ = 4). The player did not use 1.
- For ♥, ♣, ♦, ♦, 10, 2, and 1 in one roll, the player formed two addition sentences (among other possibilities): ♥ + ♣ = ♦, (where ♥ = 3, ♣ = 5, and ♦ = 8), 2 + ♦ = 10 (where ♦ = 8). The player was not able to use 1 within this move. Notice that ♦ has to have the same value for both sentences in this turn.

Students' Responses for Survivor's Game Level III

The following represent some of the addition sentences made by the students during the treatment for Survivor's Game Level III:

- For 0, 0, 1, 2, 4, 7, 5, 10, and 18 in one roll, the player formed only one addition sentence, 1 + 4 = 5. So far this move was worth three points. The player was not using 0, 0, 2, 7, 10, and 18. The player had the following options for this version of the game:
  - Rolling all dice again (including the ones used for the addition sentence).
  - Rolling some or all of the other remaining dice (not being used: 0, 0, 2, 7, 10, and 18).
  - Staying with the three points she had so far and not rolling again.
  - The above options were available for two more extra rolls in the same turn.
- For 1, 2, 4, 5, 5, 7, 7, 10, and 17, in one roll, the player formed three addition sentences: 1 + 4 = 5, 5 + 2 = 7, and 10 + 7 = 17. This move was worth nine points (one point per die or number used correctly).
- For 1, 1, 2, 2, 3, 4, 5, 5, 7, and ♣ in one roll, the player formed three addition sentences: 1 + 4 = 5, 2 + 2 = 3, and 5 + 2 = ♣, where ♣ = 7. This was worth ten points (2 points for the correct use of the symbol and one for each number used correctly).
- For ♥, ♣, ☹, ☹, 5, 1, 1, 3, and 3 in one roll, the player formed two addition sentences: ♥ + 3 = 5 (where ♥ = 2), and 3 + 1 = ♣ (where ♣ = 4). This was worth eight points. The player was not allowed to use the dice with the ☹'s. This player selected to roll the other dice (the ones without the ☹'s) again to try to earn more points during his turn.
- For ♥, ♥, ♣, ♦, ♦, 1, 9, 11, and 15 in one roll, the player formed the following three addition sentences (among the other possibilities she had): ♥ + ♣ = 11, (where ♥ = 7, and ♣ = 4), 1 + ♦ = 9 (where ♦ = 8), and ♥ + ♦ = 15. This move was worth fourteen points. Notice that ♥ and ♦ had to have the same value for both sentences in this turn. The player was not allowed to use two different values for the same symbol within a given turn. The values only could change when the dice were rolled again during a different turn.
- For 1, 1, 2, 3, 4, 5, 5, 7, and ♣ in one roll, the player formed three addition sentences: 1 + 4 = 5, 2 + 1 = 3, and 5 + 2 = ♣, where ♣ = 7. This was worth ten points (2 points for the correct use of the symbol and one for each number used correctly).
- For 0, 0, 1, 2, 4, 4, 7, 10, and ☹, in one roll, the player could not formed a correct addition sentence on the game board (two addends and a sum) with these numbers, and could not earn points for this turn. The player opted to roll all of the dice again, except for the one with the ☹).

Students' Responses for Survivor's Game 1 Level IV

The following represent some of the addition sentences made by the students during the treatment for Survivor's Game 1 Level IV:

- For 1, 2, 2, 4, 4, 5, 5, 7, 10, and 10, in one roll, the player formed three multiplication sentences: $1 \bullet 4 = 4$, $5 \bullet 2 = 10$, and $2 \bullet 5 = 10$. Seven was not used. This move was worth nine points.
- For 1, 1, 2, 2, 2, 5, 5, 5, 6, and Y in one roll, the player made three multiplication sentences: $1 \bullet 5 = 5$, $2 \bullet 1 = 2$, and $5 \bullet 2 = Y$, where $Y = 10$. Six was not used in this move. This was worth ten points (2 points for the correct use of the symbol and one for each number used correctly).
- For X, Y, ☹, ☹, 1, 2, 3, 3, 5 and 15 in one roll, the player formed two multiplication sentences: $X \bullet 3 = 15$ (where $X = 5$), and $3 \bullet 1 = Y$ (where $Y = 3$). Two and five were not used. This was worth eight points. The player was not allowed to use the dice with the ☹'s. The player would not be able to roll again since two ☹'s and one die is left to play (the dice with ☹'s were not allowed to be rolled again).
- For X, Y, Z, 4, 3, 2, 2, 1, 1, and 1 in one roll, the player formed the following two multiplication sentences (among other possibilities): $X \bullet Y = Z$ (where $X = 4$, $Y = 2$, and $Z = 8$), $2 \bullet 1 = 2$. This one was worth nine points. The player did not use 4, 3, and one of the 1's.
- For X, X, Y, Y, Z, Z, 1, 9, 28, and 63 in one roll, the player formed the following three multiplication sentences (among other possibilities): $X \bullet Y = 28$, (where $X = 7$, and $Y = 4$), $1 \bullet Z = 9$ (where $Z = 9$), and $X \bullet Z = 63$. One of the Y's was not used in this move. This move was worth fourteen points. Notice that X and Z had to have the same values for both sentences in this turn.

Students' Responses for Survivor's Game 2 Level IV

The following represent some of the addition sentences made by the students during the treatment for Survivor's Game 2 Level IV:

- For 1, 2, 2, 4, 4, 5, 5, 7, 10, and 10, in one roll, the player formed three division sentences: $4 \div 1 = 4$, $10 \div 2 = 5$, and $10 \div 5 = 2$. Seven was not used. This move was worth nine points.
- For X, Y, ☹, ☹, 1, 2, 3, 3, 5 and 15 in one roll, the player formed two division sentences: $15 \div X = 3$ (where $X = 5$), and $3 \div 1 = Y$ (where $Y = 3$). Two and five

were not used. This was worth eight points. The player was not allowed to use the two dice with the ☹'s. The player could have selected to roll the other dice (the ones without the ☹'s) again if he wanted to try to earn more points during this turn.

Discussion

After taking a closer look at means for Pre-test and Post-test scores for the Basic Facts by grade levels, the researcher found an average or above average mastery of these facts by most of the grade levels. These findings suggest that the students had little room for major improvement in this area, and that this mastery was maintained after the treatment. This also implies that students' performance in terms of algebraic thinking was not limited by their mastery of the operation basic facts. The means for the Pre-test scores for Algebraic Thinking by grade levels were low for Kindergarten, and first-grade, and from average to above average for the other remaining grades.

The Paired Sample T-test for the Pre-test and Post-test for the Addition Basic Facts for Kindergarten indicate a significant difference at this level (see table 2). This suggests that playing the game had a positive effect on Kindergarten students' mastery of the addition basic facts. Similarly, this was also true for first-grade students. A high statistically significant difference was found for first-grade students for the Paired Sample T-test involving the Pre-test and Post-test scores for Algebraic Thinking. This finding indicates an average gain of four points in this area, and suggests that second graders were positively affected by playing the addition game. Most of the students' performance improvements were related to items like the following:

$2 + 1 = \clubsuit, \clubsuit =$ ____;

$2 + 2 = \heartsuit, \heartsuit =$ ___; and

$2 + 0 = \diamondsuit, \diamondsuit =$ ____.

The correct expected answers for these items were the following: $\clubsuit = 3$, $\heartsuit = 4$, and $\diamondsuit = 2$ (this one involved the use of zero as the identity element), respectively.

In this same area for second-grade, the researcher found some improvement in the students' scores but not statistically significant. The results were not significant for the other grades in terms of basic facts or algebraic thinking for either the pre-tests or post-tests. At these grade levels, students maintained similar performance levels for pre-test or post-test.

The Pre-test and Post-test for Algebraic Thinking involving multiplication given to third-, fourth-, and fifth-graders presented some difficulties regarding students' performance in some of the items. First, the following two items indicated a lack of students' use of different value possibilities for a variable:

$Z \bullet Y = Z$, and

$X \bullet X = Z$, $X =$ ___, $Z =$ ____.

The expected correct answers for the first item were Z = any number and Y = 1, or Z = 0 and Y = 0, and for the second one was X = any number and $Z = X^2$. The first item involved the generalization of using 1 as the multiplication identity element and the second one the use of square numbers. Most students selected one possible value as a possible answer for either of

these items (for example, Z = 3, Y = 1, for the first one, and X = 3, Z = 9 for the second one). It would have been better if students indicated the possibility of different answers but none of them were able to do this. This type of thinking would have indicated flexibility in the use of the variables.

Second, most students in third-, fourth- and fifth-grades also had difficulties with the following two items:

$4 \bullet 3 = Y \bullet 6$, Y = _____; and

$Z \bullet 8 = Y \bullet 6$, Y = _____ Z = _____.

The expected correct answer for the first item was Y = 2. The most frequent incorrect answer was Y = 12. The students thought about this problem as $4 \bullet 3 = Y$, where Y was equal to 12 and last part of the equation ($\bullet 6$) was disregarded. This might indicate a lack of familiarity with this type of notation or lack of understanding of proportional thinking. This aspect needs to be investigated further in the future. For the other one, the expected correct answers were the following:

Y = 0, Z = 0,

Y = 4, Z = 3, or

Y = 8, Z = 6.

As before, it would have been better if students indicated the possibility of different answers, and lack of familiarity could have been a problem. This situation indicates the students' need to understand the use variables as a placeholder in an expression or equation. The students were

supposed to explore the role of Y in the equation $4 \bullet 3 = Y \bullet 6$ and be able to find the value of Y that makes the equation true (NCTM, 2000). Similarly to the Survivor's Game Level III (see fig. 5), this type of exploration could be provided for multiplication by adapting the Survivor's Game 1 Level IV (see fig. 8), and creating the Survivor's Game 3 Level IV (see fig. 11). More experimentation is needed in this area.

Most of the correlation coefficients were significant by grade levels. The most relevant one is the one between the Pre-test scores for Basic Facts and the Post-test scores for Algebraic Thinking. This suggests a relationship between mastery of basic facts and performance development of algebraic thinking as presented in this study.

General Observations

It was noticed during students' participation in the study that some basic prerequisites were needed to play the games effectively. The following concepts and skills were found to be important: rational counting (also know as meaningful counting), numeral recognition, number/numeral matching, operation concepts (for the specific game being played), and using of the operation (for the specific game being played), and equal signs (+, •, ÷, and =).

Furthermore, in terms of rational counting, it is important to point out that the number positions on the dice for each game were predetermined in order to limit the possible sums, products or quotients. This also allowed student to use the symbols as variables that represent values they could handle within a given level of the game, and limited the domain and range of the variables for most combinations. Note that, in some situations, the students could form

addition or multiplication sentences with sums higher that 18 or products higher that 81. For example, ♣ + 5 = ♥, where ♣ = 100, and ♥ = 105.

Having these prerequisites already mastered facilitated students' understanding of the game and the development of mathematical strategies. Place value knowledge, memorization of the addition basic facts, and numeral writing were not required for these games. However, the significant correlation between students' performance in the post-tests involving addition basic facts and the post-tests involving algebraic thinking seems to indicated that it is beneficial to master the basic facts in order to develop algebraic thinking and make generalizations (Usiskin, 1988, 1997).

Conclusion

The use of these games provided plenty of practice for addition, multiplication, or division basic facts in a motivational and challenging environment. The students challenged and encouraged each other as they played these games. The games were especially effective for Kindergarten, first- and second-grades. The early introduction to some ideas related to algebra was possible and a very important one for these students. It is recommended that students' understanding of algebraic thinking be explored further, and to take a closer look at the students' levels of understanding at different stages.

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Figure 1

Game Board for Survivor's Level I (For Sums Up to 5)

Place one die on each blank to form an addition sentence:

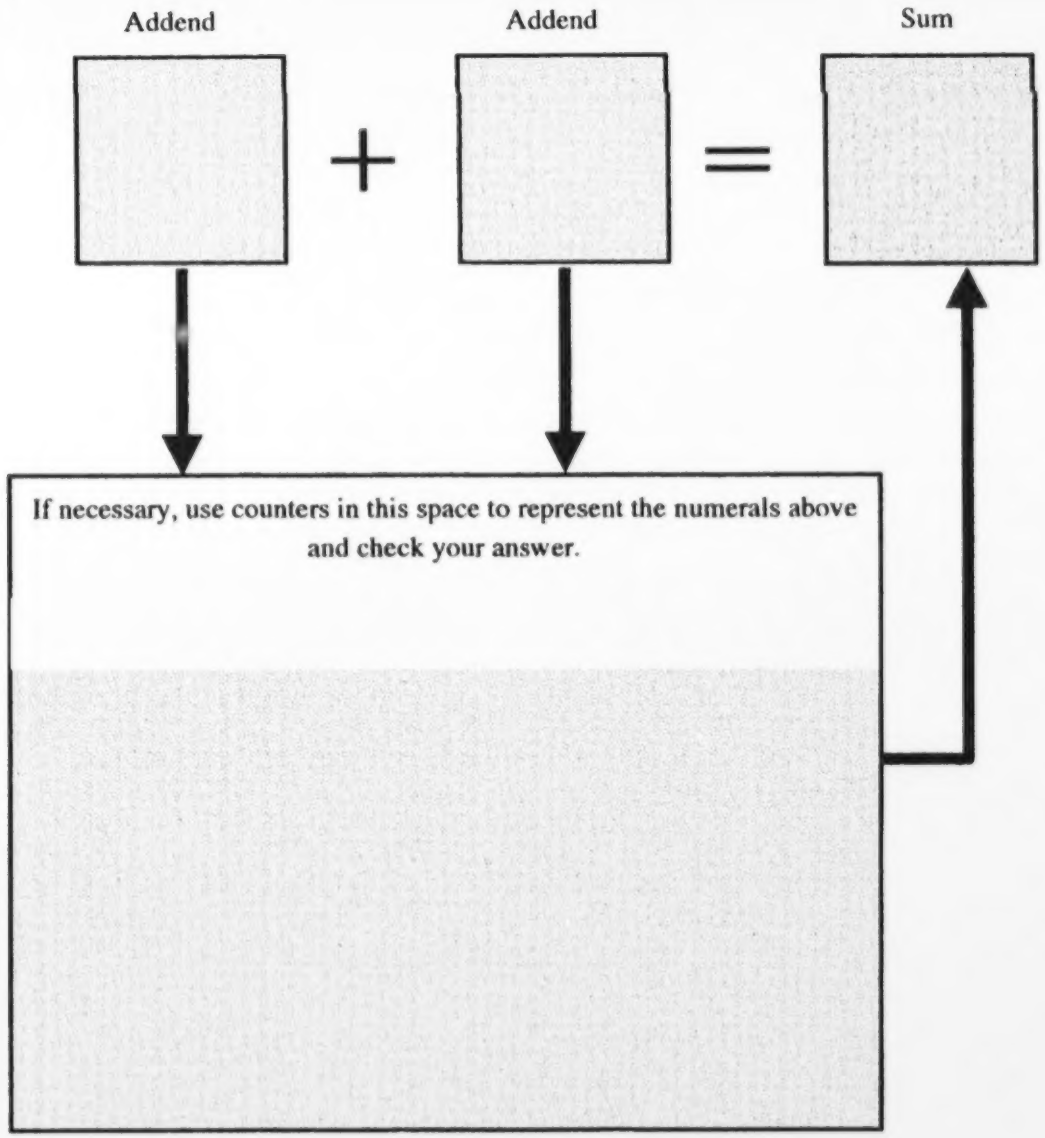

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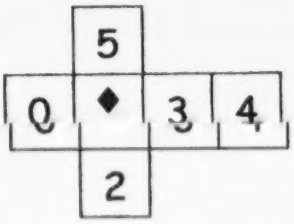
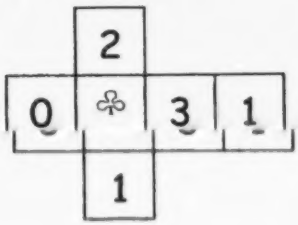
Figure 2

Dice for Survivor's Game Level I

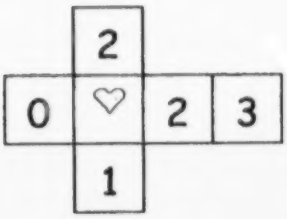
Die 1



Die 2



Die 3



Die 4

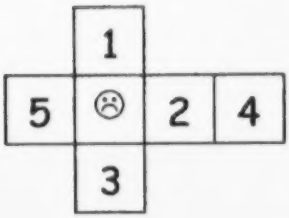


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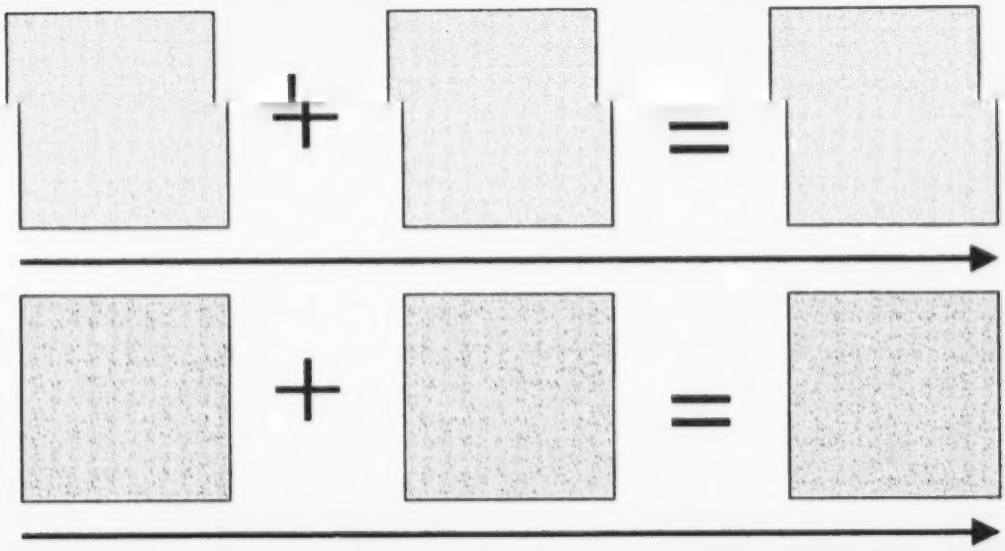
Figure 3

Game Board for Survivor's Level II (For Sums Up to 10)

Place one die on each blank to form an addition sentence:



If necessary, use counters in this space to represent the numerals above and check your answer.

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Figure 4

Dice for Survivor's Game Level II

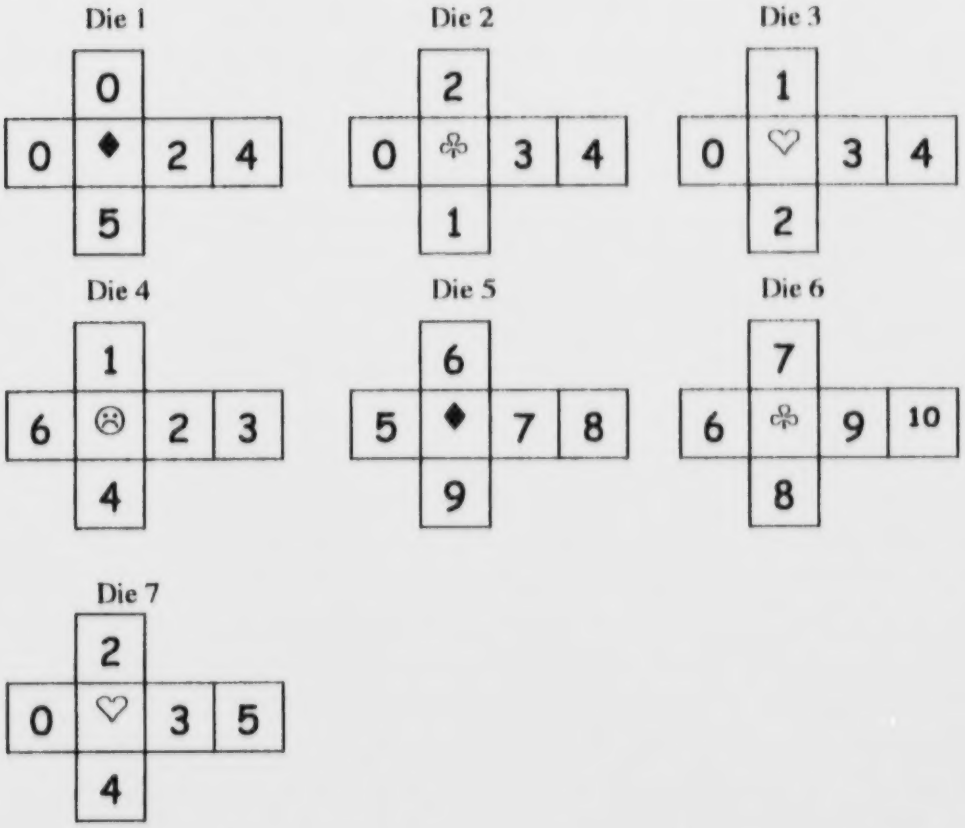

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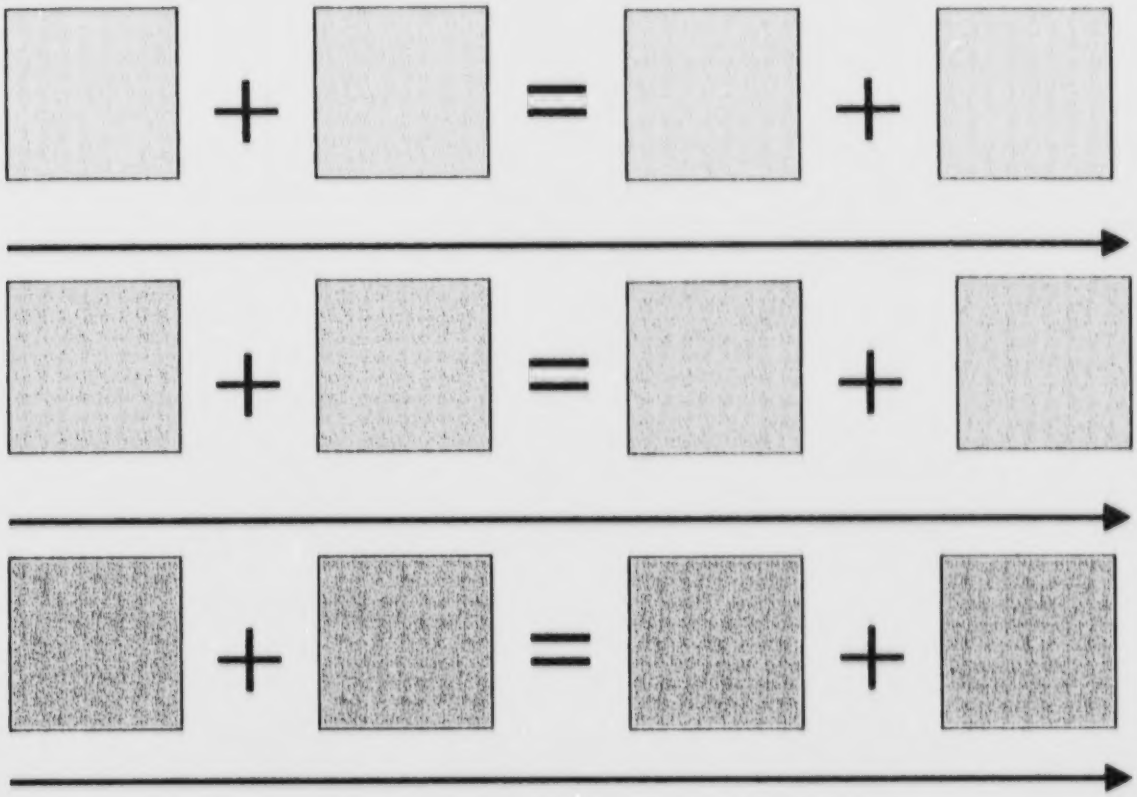
Figure 5

Game Board for Survivor's Game Level III (For Sums Up to 18)

Place one die on each blank to form correct multiplication sentences.



Dice with a ☹ that can not be played in this game. Place each die with ☹ here. They may not be played until next round or game.

Die 1 Die 2 Die 3 Die 4

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Figure 6

Dice for Survivor's Game Level III

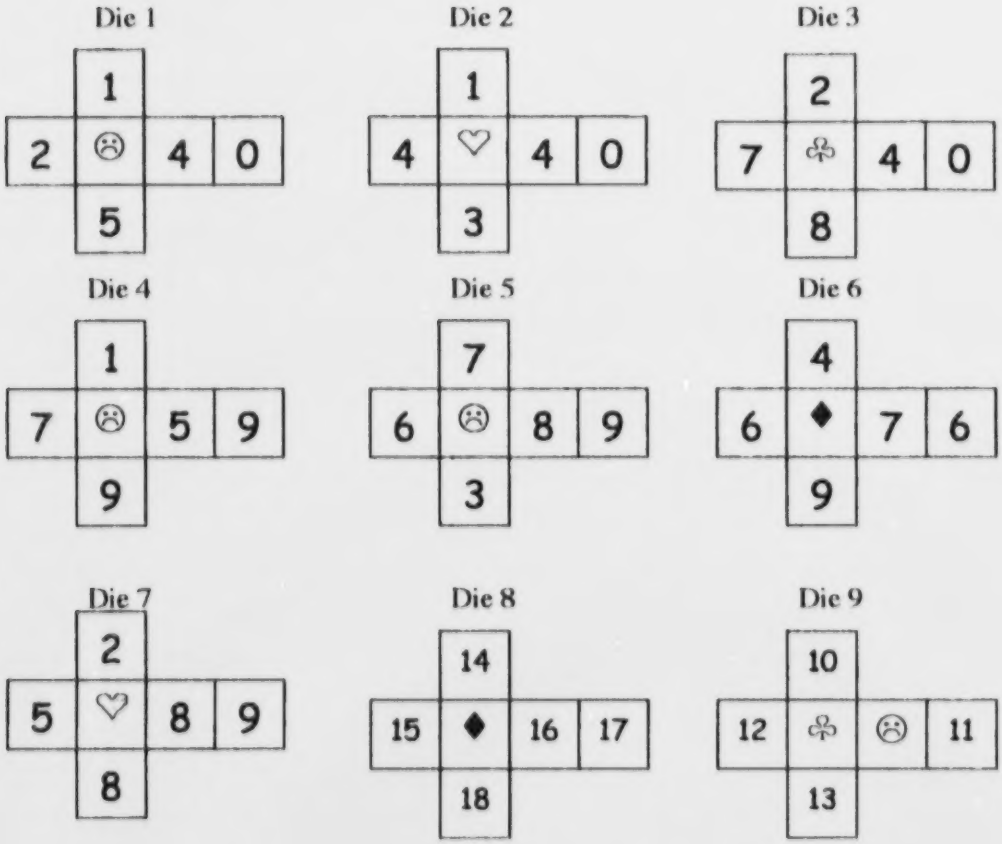

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Figure 7

Recording Form for Survivor's Game Levels III and IV

Points Per Player

Name	Round 1 Points	Round 2 Points	Round 3 Points	Total Points for All Three Rounds
1.				
2.				
3.				
4.				

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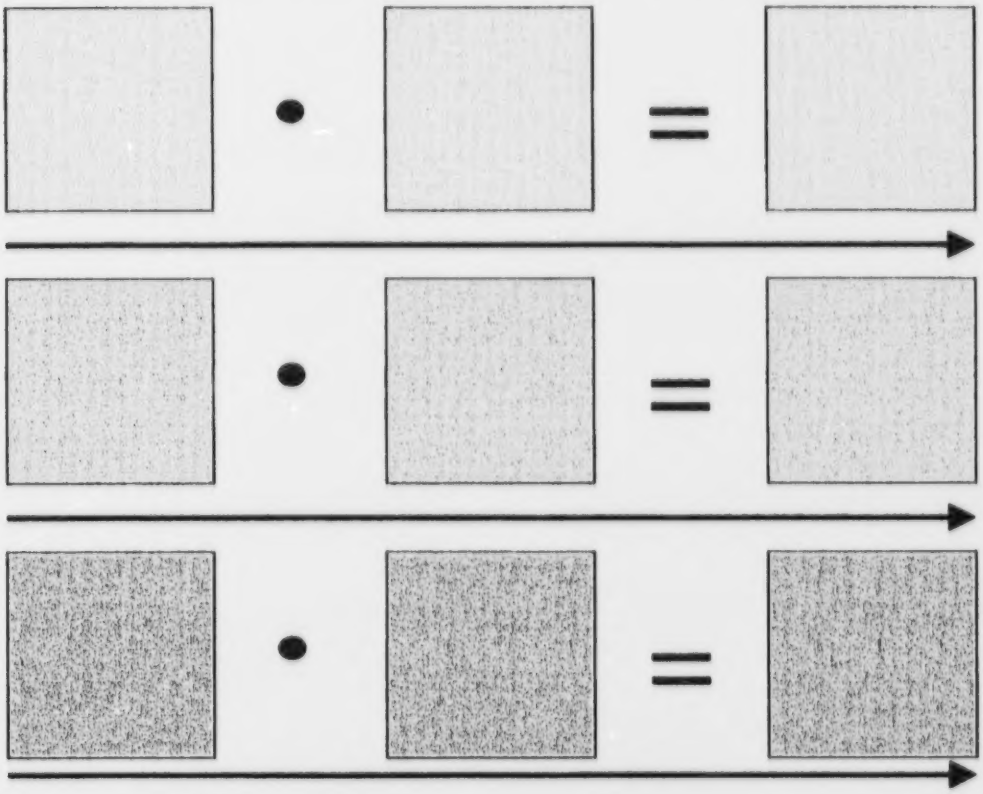
Figure 8

Game Board for Survivor's Game 3 Level IV (For Products Up to 81)

Place one die on each blank to form correct multiplication sentences.



Dice with a ☹ that can not be played in this game. Place each die with ☹ here. They may not be played until next round or game.

Die 1 Die 2 Die 3 Die 4

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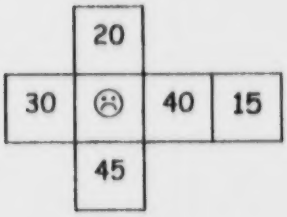
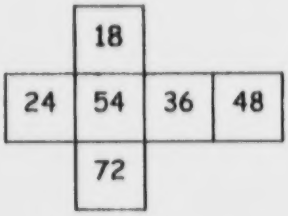
Figure 9

Dice for Survivor's Games 1, 2 or 3 Level IV (For Products and Quotients from 0 to 81)

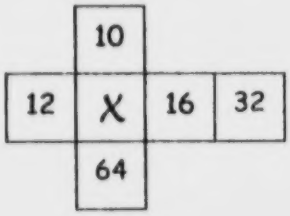
Die 1



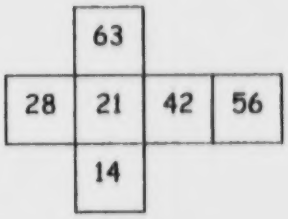
Die 2



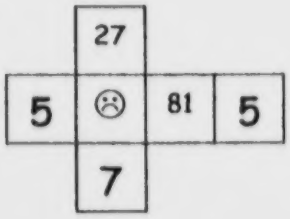
Die 3



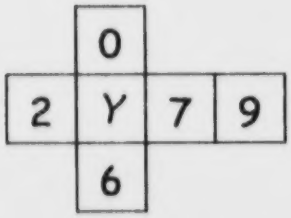
Die 4



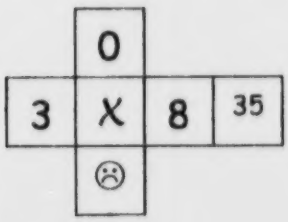
Die 5



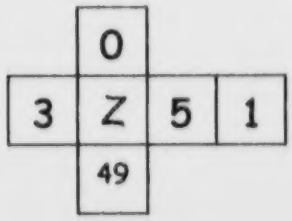
Die 6



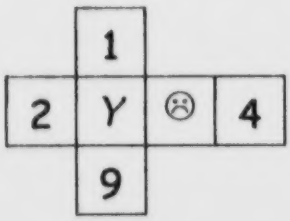
Die 7



Die 8



Die 9



Die 10

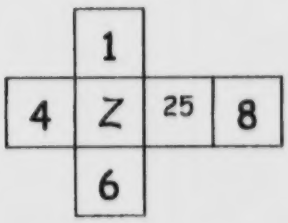


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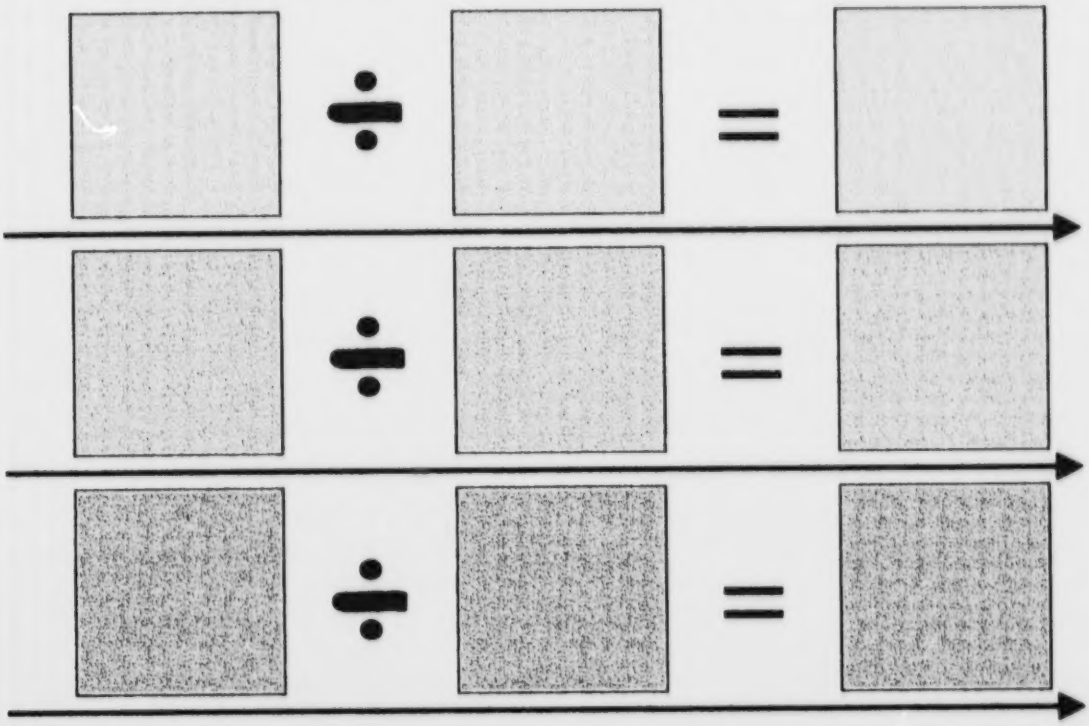
Figure 10

Game Board for Survivor's Game 2 Level IV (For Quotients from 0 to 81)

Place one die on each blank to form correct division sentences.



Dice with a ☹ that can not be played in this game. Place each die with ☹ here. They may not be played until next round or game.

Die 1 Die 2 Die 3 Die 4

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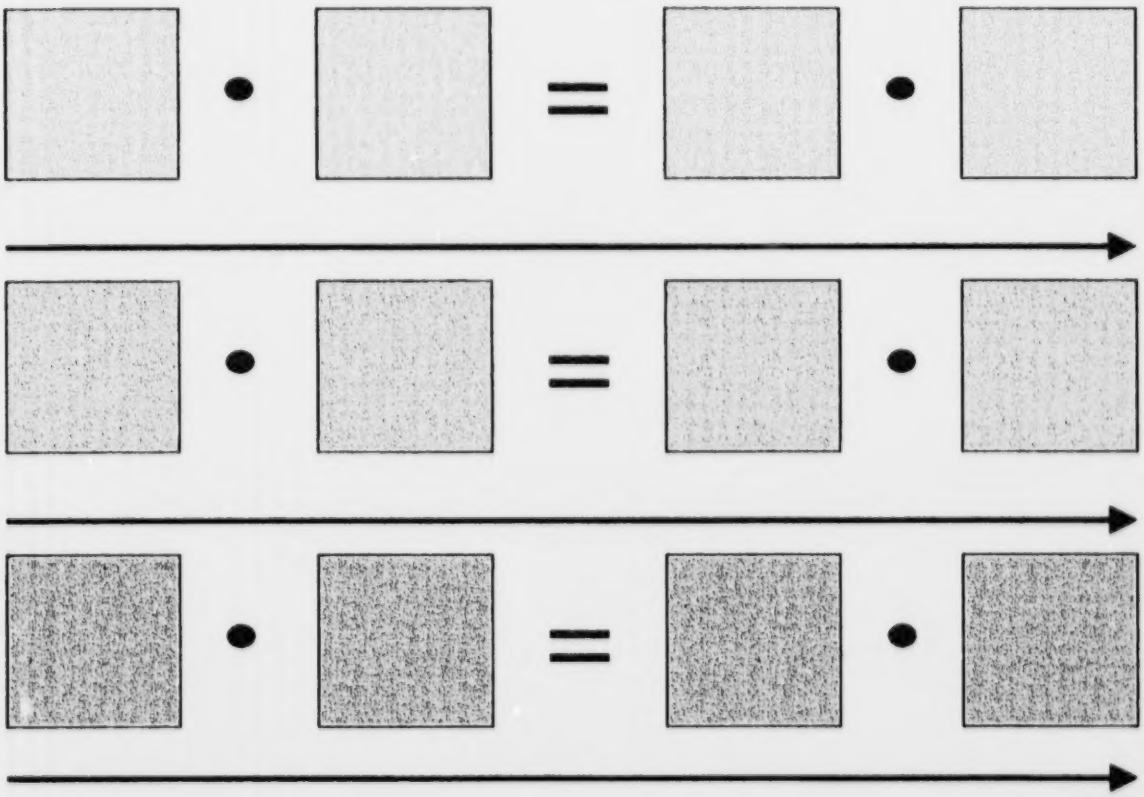
Figure 11

Game Board for Survivor's Game 3 Level IV

(For Multiplication Expressions on Both Sides)

Place one die on each blank to form correct multiplication sentences.



Dice with a ☹ that can not be played in this game. Place each die with ☹ here. They may not be played until next round or game.

Die 1 Die 2 Die 3 Die 4

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Table 1

Descriptive Statistics by Grade

Grade	Test	Mean	N	St. Dev.	St. Error Mean	Highest Possible Score
Kinder	Pre-test ABF Level I	14.50	34	6.73	1.15	21
	Post-test ABF Level I	17.26	34	6.51	1.12	21
	Pre-test AAT Level I	2.21	34	3.22	0.55	10
	Post-test AAT Level I	3.09	34	3.35	0.57	10
First	Pre-test ABF Level II	60.96	24	6.24	1.27	64
	Post-test ABF Level II	62.00	24	6.32	1.29	64
	Pre-test AAT Level II	1.79	24	4.32	0.88	18
	Post-test AAT Level II	7.08	24	5.97	1.22	18
Second	Pre-test ABF Level III	97.05	19	3.58	0.82	100
	Post-test ABF Level III	96.79	19	4.34	1.00	100
	Pre-test AAT Level II	11.89	19	4.20	0.96	18
	Post-test AAT Level II	13.79	19	4.81	1.10	18
Third	Pre-test MBF Level IV	81.63	24	26.71	5.45	100
	Post-test MBF Level VI	86.96	24	17.93	3.66	100
	Pre-test MAT Level III	12.5	24	5.33	1.09	18
	Post-test MAT Level III	13.50	24	4.95	1.01	18
Fourth	Pre-test MBF Level IV	96.59	17	5.39	1.31	100
	Post-test MBF Level IV	96.06	17	4.72	1.15	100
	Pre-test MAT Level III	14.41	17	2.40	0.58	18
	Post-test MAT Level III	13.94	17	3.15	0.76	18
Fifth	Pre-test MDBF Level IV	169.04	23	36.96	7.71	190
	Post-test MDBF Level IV	170.70	23	35.06	7.31	190
	Pre-test MDAT Level III	27.43	23	6.85	1.43	34
	Post-test MDAT Level III	28.13	23	6.66	1.39	34

Note: **ABF** = Addition Basic Facts,
MBF = Multiplication Basic Facts,
MDAB = Multiplication and Division Basic Facts,
AAT = Addition Algebraic Thinking,
MAT = Multiplication Algebraic Thinking,
MDAT = Multiplication and Division Algebraic Thinking.

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Table 2

Paired Sample T-test by Grade Level

Grade	Pair	Mean	Std. Dev.	Std. Error Mean	t	df	Sig. (2-tailed)
Kinder	Pre-test ABF - Post-test ABF	-2.76	4.90	0.84	-3.290	33	0.002
	Pre-test AAT - Post-test AAT	-0.88	4.13	0.71	-1.245	33	0.222
First	Pre-test ABF - Post-test ABF	-1.04	1.68	0.34	-3.037	23	0.006
	Pre-test AAT - Post-test AAT	-5.29	5.46	1.11	-4.750	23	.000
Second	Pre-test ABF - Post-test ABF	0.26	3.26	0.75	0.32	18	0.729
	Pre-test ABF - Post-test ABF	-1.89	4.03	0.92	-2.051	18	0.055
Third	Pre-test MBF - Post-test MBF	-5.33	15.25	3.11	-1.714	23	0.100
	Pre-test MAT - Post-test MAT	-1.00	3.15	0.64	-1.556	23	0.133
Fourth	Pre-test MBF - Post-test MBF	0.53	4.36	1.06	0.501	16	0.623
	Pre-test MAT - Post-test MAT	0.47	3.08	0.75	0.629	16	0.538
Fifth	Pre-test MDBF - Post-test MDBF	-1.65	7.84	1.63	-1.011	22	0.323
	Pre-test MDAT - Post-test MDAT	-0.70	3.30	0.69	-1.012	22	0.322

Note: **ABF** = Addition Basic Facts,
MBF = Multiplication Basic Facts,
MDAB = Multiplication and Division Basic Facts,
AAT = Addition Algebraic Thinking,
MAT = Multiplication Algebraic Thinking,
MDAT = Multiplication and Division Algebraic Thinking.

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Table 3

Pearson Correlation Coefficient By Grade Level

Grade Level	Pre-test BF - Post-test BF	Post-test BF - Post-test AT	Pre-test AT - Post-test AT
Kinder	0.726**	0.331	0.209
First	0.964**	0.311	0.476*
Second	0.676**	0.351	0.609**
Third	0.838**	0.841**	0.815**
Fourth	0.635**	0.659**	0.408
Fifth	0.978**	0.889**	0.881**

Note: **BF** = Basic Facts, **AT** = Algebraic Thinking

** = Correlation is significant at the 0.01 level (2-tailed)

* = Correlation is significant at the 0.05 level (2-tailed)

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Appendix

Instruments

Pre-test Addition Basic Facts for Sums Up to 5: Level I

Find the sum:

1 + 2 = ___	1 + 1 = ___	2 + 2 = ___	2 + 1 = ___
2 + 0 = ___	1 + 3 = ___	0 + 5 = ___	1 + 0 = ___
1 + 4 = ___	0 + 2 = ___	3 + 0 = ___	0 + 0 = ___
3 + 2 = ___	4 + 0 = ___	0 + 3 = ___	4 + 1 = ___
0 + 4 = ___	0 + 1 = ___	2 + 3 = ___	3 + 1 = ___
5 + 0 = ___			

Pre-test Algebraic Thinking Involving Addition Level I

Find the number: (Note: ♡, ♧, ♦ equal to any number.)

Example 1: 3 + 2 = ♧ ♧ = 5	Example 2 2 + ♡ = 3 ♡ = 1 (Because 2 + 1 = 3.)	Example 3 ♦ + ♡ = 3 ♦ = 2 ♡ = 1 (Because 2 + 1 = 3.)	
2 + 1 = ♧ ♧ = ____	2 + 2 = ♡ ♡ = ____	2 + 0 = ♦ ♦ = ____	3 + ♦ = 3 ♦ = ____
♧ + 1 = 3 ♧ = ____	1 + ♡ = 4 ♡ = ____	♡ + ♧ = 4 ♡ = 3 ♧ = ____	♡ + 4 = ♦ ♡ = ____ ♦ = 5
♡ + ♧ = 4 ♡ = 2 ♧ = ____	♡ + ♧ = ♦ ♡ = 1 ♧ = ____ ♦ = 3		

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Pre-test Addition Basic Facts for Sums Up to 10: Level II

Find the sum:

1. 1 + 2 = ___	2. 1 + 1 = ___	3. 2 + 2 = ___	4. 2 + 1 = ___	5. 0 + 1 = ___
6. 2 + 0 = ___	7. 1 + 3 = ___	8. 0 + 5 = ___	9. 1 + 0 = ___	10. 2 + 3 = ___
11. 1 + 4 = ___	12. 0 + 2 = ___	13. 3 + 0 = ___	14. 0 + 0 = ___	15. 3 + 1 = ___
16. 3 + 2 = ___	17. 4 + 0 = ___	18. 0 + 3 = ___	19. 4 + 1 = ___	20. 5 + 0 = ___
21. 0 + 4 = ___	22. 4 + 4 = ___	23. 3 + 5 = ___	24. 4 + 3 = ___	25. 2 + 6 = ___
26. 4 + 6 = ___	27. 8 + 2 = ___	28. 8 + 0 = ___	29. 4 + 5 = ___	30. 5 + 1 = ___
31. 8 + 1 = ___	32. 6 + 0 = ___	33. 2 + 8 = ___	34. 2 + 7 = ___	35. 5 + 5 = ___
36. 6 + 1 = ___	37. 1 + 7 = ___	38. 1 + 8 = ___	39. 1 + 9 = ___	40. 2 + 4 = ___
41. 5 + 4 = ___	42. 9 + 1 = ___	43. 0 + 9 = ___	44. 1 + 5 = ___	45. 6 + 3 = ___
46. 3 + 3 = ___	47. 3 + 7 = ___	48. 6 + 2 = ___	49. 1 + 6 = ___	50. 7 + 0 = ___
51. 5 + 3 = ___	52. 7 + 1 = ___	53. 7 + 2 = ___	54. 6 + 4 = ___	55. 2 + 5 = ___
56. 7 + 3 = ___	57. 0 + 8 = ___	58. 3 + 4 = ___	59. 0 + 7 = ___	60. 9 + 0 = ___
61. 5 + 2 = ___	62. 4 + 2 = ___	63. 0 + 6 = ___	64. 3 + 6 = ___	

Pre-test Algebraic Thinking Involving Addition Level II

Find the number: (Note: ♡, ♧, ♦ equal to any number.)

Example 1:	Example 2:	Example 3:	Example 4:
3 + 2 = ♧ ♧ = 5	1 + ♡ = 3 ♡ = 2 (Because 1 + 2 = 3.)	2 + ♡ = ♦ ♡ = 2 ♦ = 4 (Because 2 + 2= 1.)	♦ + ♦ = ♡ ♦ = 1 ♡ = 2 (Because 1 + 1 = 2.)
1. 2 + 5 = ♧ ♧ = ____	2. 4 + 4 = ♡ ♡ = ____	3. 6 + 0 = ♦ ♦ = ____	4. 5 + ♧ = 5 ♧ = ____
5. ♧ + 5 = 7 ♧ = ____	6. 1 + ♡ = 8 ♡ = ____	7. ♡ + ♧ = 4 ♡ = 3 ♧ = ____	8. ♡ + ♦ = 4 ♡ = 1 ♦ = ____
9. ♡ + 4 = ♦ ♡ = ____ ♦ = 6	10. ♧ + ♧ = 8 ♧ = ____	11. ♡ + 0 = ♡ ♡ = ____	12. ♦ + ♦ = 6 ♦ = ____
13. ♡ + ♡ = ♦ ♡ = ___ ♦ = ___	14. ♦ + ♧ = ♦ ♦ = 3 ♧ = ____	15. ♡ + ♧ = ♧ ♡ = ____ ♧ = 5	16. ♡ + ♧ = ♦ ♡ = 1 ♧ = ____ ♦ = 5
17. ♦ + ♧ = ♦ ♦ = ____ ♧ = ____	18. ♡ + ♧ = ♧ ♡ = ____ ♧ = ____		

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Pre-test Addition Basic Facts for Sums Up to 18: Level III

Find the sum of each:

1. 1 + 2 = ___	2. 1 + 1 = ___	3. 2 + 2 = ___	4. 2 + 1 = ___	5. 0 + 1 = ___
6. 2 + 0 = ___	7. 1 + 3 = ___	8. 0 + 5 = ___	9. 1 + 0 = ___	10. 2 + 3 = ___
11. 1 + 4 = ___	12. 0 + 2 = ___	13. 3 + 0 = ___	14. 0 + 0 = ___	15. 3 + 1 = ___
16. 3 + 2 = ___	17. 4 + 0 = ___	18. 0 + 3 = ___	19. 4 + 1 = ___	20. 5 + 0 = ___
21. 0 + 4 = ___	22. 4 + 4 = ___	23. 3 + 5 = ___	24. 4 + 3 = ___	25. 2 + 6 = ___
26. 4 + 6 = ___	27. 8 + 2 = ___	28. 8 + 0 = ___	29. 4 + 5 = ___	30. 5 + 1 = ___
31. 8 + 1 = ___	32. 6 + 0 = ___	33. 2 + 8 = ___	34. 2 + 7 = ___	35. 5 + 5 = ___
36. 6 + 1 = ___	37. 1 + 7 = ___	38. 1 + 8 = ___	39. 1 + 9 = ___	40. 2 + 4 = ___
41. 5 + 4 = ___	42. 9 + 1 = ___	43. 0 + 9 = ___	44. 1 + 5 = ___	45. 6 + 3 = ___
46. 3 + 3 = ___	47. 3 + 7 = ___	48. 6 + 2 = ___	49. 1 + 6 = ___	50. 7 + 0 = ___
51. 5 + 3 = ___	52. 7 + 1 = ___	53. 7 + 2 = ___	54. 6 + 4 = ___	55. 2 + 5 = ___
56. 7 + 3 = ___	57. 0 + 8 = ___	58. 3 + 4 = ___	59. 0 + 7 = ___	60. 9 + 0 = ___
61. 5 + 2 = ___	62. 4 + 2 = ___	63. 0 + 6 = ___	64. 3 + 6 = ___	65. 2 + 9 = ___
66. 4 + 9 = ___	67. 5 + 6 = ___	68. 7 + 5 = ___	69. 9 + 5	70. 6 + 5 = ___
71. 5 + 8 = ___	72. 3 + 8 = ___	73. 9 + 2 = ___	74. 9 + 9 = ___	75. 7 + 9 = ___
76. 7 + 7 = ___	77. 8 + 5 = ___	78. 4 + 7 = ___	79. 6 + 6 = ___	80. 9 + 3 = ___
81. 8 + 6 = ___	82. 6 + 8 = ___	83. 8 + 3 = ___	84. 7 + 4 = ___	85. 8 + 8 = ___
86. 4 + 8 = ___	87. 7 + 8 = ___	88. 3 + 9 = ___	89. 8 + 9 = ___	90. 5 + 7 = ___
91. 8 + 7 = ___	92. 9 + 7 = ___	93. 8 + 4 = ___	94. 7 + 6 = ___	95. 9 + 4 = ___
96. 9 + 6 = ___	97. 6 + 7 = ___	98. 6 + 9 = ___	99. 5 + 9 = ___	100. 9 + 8 = ___

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Pre-test Multiplication Basic Facts for Products Up to 81: Level IV

Find the product of each: (Note: • means multiply)

1. 1 • 2 = ___	2. 1 • 1 = ___	3. 2 • 2 = ___	4. 2 • 1 = ___	5. 0 • 1 = ___
6. 2 • 0 = ___	7. 1 • 3 = ___	8. 0 • 5 = ___	9. 1 • 0 = ___	10. 2 • 3 = ___
11. 1 • 4 = ___	12. 0 • 2 = ___	13. 3 • 0 = ___	14. 0 • 0 = ___	15. 3 • 1 = ___
16. 3 • 2 = ___	17. 4 • 0 = ___	18. 0 • 3 = ___	19. 4 • 1 = ___	20. 5 • 0 = ___
21. 0 • 4 = ___	22. 4 • 4 = ___	23. 3 • 5 = ___	24. 4 • 3 = ___	25. 2 • 6 = ___
26. 4 • 6 = ___	27. 8 • 2 = ___	28. 8 • 0 = ___	29. 4 • 5 = ___	30. 5 • 1 = ___
31. 8 • 1 = ___	32. 6 • 0 = ___	33. 2 • 8 = ___	34. 2 • 7 = ___	35. 5 • 5 = ___
36. 6 • 1 = ___	37. 1 • 7 = ___	38. 1 • 8 = ___	39. 1 • 9 = ___	40. 2 • 4 = ___
41. 5 • 4 = ___	42. 9 • 1 = ___	43. 0 • 9 = ___	44. 1 • 5 = ___	45. 6 • 3 = ___
46. 3 • 3 = ___	47. 3 • 7 = ___	48. 6 • 2 = ___	49. 1 • 6 = ___	50. 7 • 0 = ___
51. 5 • 3 = ___	52. 7 • 1 = ___	53. 7 • 2 = ___	54. 6 • 4 = ___	55. 2 • 5 = ___
56. 7 • 3 = ___	57. 0 • 8 = ___	58. 3 • 4 = ___	59. 0 • 7 = ___	60. 9 • 0 = ___
61. 5 • 2 = ___	62. 4 • 2 = ___	63. 0 • 6 = ___	64. 3 • 6 = ___	65. 2 • 9 = ___
66. 4 • 9 = ___	67. 5 • 6 = ___	68. 7 • 5 = ___	69. 9 • 5	70. 6 • 5 = ___
71. 5 • 8 = ___	72. 3 • 8 = ___	73. 9 • 2 = ___	74. 9 • 9 = ___	75. 7 • 9 = ___
76. 7 • 7 = ___	77. 8 • 5 = ___	78. 4 • 7 = ___	79. 6 • 6 = ___	80. 9 • 3 = ___
81. 8 • 6 = ___	82. 6 • 8 = ___	83. 8 • 3 = ___	84. 7 • 4 = ___	85. 8 • 8 = ___
86. 4 • 8 = ___	87. 7 • 8 = ___	88. 3 • 9 = ___	89. 8 • 9 = ___	90. 5 • 7 = ___
91. 8 • 7 = ___	92. 9 • 7 = ___	93. 8 • 4 = ___	94. 7 • 6 = ___	95. 9 • 4 = ___
96. 9 • 6 = ___	97. 6 • 7 = ___	98. 6 • 9 = ___	99. 5 • 9 = ___	100. 9 • 8 = ___

Pre-test Algebraic Thinking Involving Multiplication Level IV

Find the number: (Note: X, Y, and Z are equal to any number.)

Example 1:	Example 2:	Example 3:	Example 4:
3 • 2 = X X = 6	3 • Y = 6 Y = 2 (Because 3 • 2 = 6.)	4 • X = Z X = 2 Z = 8 (Because 4 • 2 = 8.)	Z • Z = X Z = 2 X = 4 (Because 2 • 2 = 4.)
1. 2 • 5 = X X = ___	2. 4 • 4 = Z Z = ___	3. 6 • 0 = Y Y = ___	4. 5 • Z = 5 Z = ___
5. Z • 5 = 30 Z = ___	6. 1 • X = 8 X = ___	7. Z • Z = 25 Z = ___	8. X • 0 = X X = ___
9. X • 4 = Z X = ___ Z = 16	10. X • Y = 12 X = 2 Y = ___	11. X • Y = 15 X = ___ Y = 3	12. Z • Y = Z Z = 3 Y = ___
13. X • X = Z X = 3 Z = ___	14. X • Y = Z X = 1 Y = ___ Z = 5	15. Z • Y = Z Z = ___ Y = ___	16. X • X = Z X = ___ Z = ___
17. 4 • 3 = Y • 6 Y = ___	18. Z • 8 = Y • 6 Y = ___ Z = ___		

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Pre-test Division Basic Facts for Quotients Up to 81: Level IV

Divide:

1. 2 ÷ 2 = ___	2. 1 ÷ 1 = ___	3. 4 ÷ 2 = ___	4. 2 ÷ 1 = ___	5. 0 ÷ 1 = ___
6. 3 ÷ 3 = ___	7. 0 ÷ 2 = ___	8. 0 ÷ 5 = ___	9. 4 ÷ 1 = ___	10. 6 ÷ 3 = ___
11. 4 ÷ 4 = ___	12. 16 ÷ 4 = ___	13. 0 ÷ 3 = ___	14. 12 ÷ 3 = ___	15. 3 ÷ 1 = ___
16. 6 ÷ 2 = ___	17. 16 ÷ 2 = ___	18. 15 ÷ 5 = ___	19. 20 ÷ 5 = ___	20. 12 ÷ 6 = ___
21. 0 ÷ 4 = ___	22. 7 ÷ 7 = ___	23. 16 ÷ 8 = ___	24. 14 ÷ 7 = ___	25. 5 ÷ 1 = ___
26. 24 ÷ 6 = ___	27. 9 ÷ 1 = ___	28. 8 ÷ 8 = ___	29. 9 ÷ 9 = ___	30. 25 ÷ 5 = ___
31. 8 ÷ 1 = ___	32. 21 ÷ 7 = ___	33. 0 ÷ 9 = ___	34. 5 ÷ 5 = ___	35. 8 ÷ 4 = ___
36. 6 ÷ 1 = ___	37. 7 ÷ 1 = ___	38. 12 ÷ 2 = ___	39. 6 ÷ 6 = ___	40. 18 ÷ 3 = ___
41. 20 ÷ 4 = ___	42. 0 ÷ 8 = ___	43. 14 ÷ 2 = ___	44. 24 ÷ 4 = ___	45. 10 ÷ 5 = ___
46. 9 ÷ 3 = ___	47. 8 ÷ 2 = ___	48. 12 ÷ 4 = ___	49. 0 ÷ 7 = ___	50. 18 ÷ 9 = ___
51. 15 ÷ 3 = ___	52. 30 ÷ 6 = ___	53. 0 ÷ 6 = ___	54. 18 ÷ 6 = ___	55. 30 ÷ 5 = ___
56. 21 ÷ 3 = ___	57. 24 ÷ 8 = ___	58. 35 ÷ 5 = ___	59. 45 ÷ 5	60. 63 ÷ 9 = ___
61. 10 ÷ 2 = ___	62. 40 ÷ 5 = ___	63. 18 ÷ 2 = ___	64. 81 ÷ 9 = ___	65. 27 ÷ 3 = ___
66. 36 ÷ 9 = ___	67. 48 ÷ 8 = ___	68. 28 ÷ 7 = ___	69. 36 ÷ 6 = ___	70. 64 ÷ 8 = ___
71. 40 ÷ 8 = ___	72. 56 ÷ 8 = ___	73. 24 ÷ 3 = ___	74. 28 ÷ 4 = ___	75. 35 ÷ 7 = ___
76. 49 ÷ 7 = ___	77. 63 ÷ 7 = ___	78. 27 ÷ 9 = ___	79. 72 ÷ 9 = ___	80. 36 ÷ 4 = ___
81. 48 ÷ 6 = ___	82. 42 ÷ 7 = ___	83. 32 ÷ 4 = ___	84. 42 ÷ 6 = ___	85. 72 ÷ 8 = ___
86. 32 ÷ 8 = ___	87. 56 ÷ 7 = ___	88. 54 ÷ 9 = ___	89. 45 ÷ 9 = ___	90. 54 ÷ 6 = ___

Pre-test Algebraic Thinking Involving Division Level IV

Find the number: (Note: X, Y, and Z are equal to any number.)

Example 1:	Example 2:	Example 3:	Example 4:
6 ÷ 2 = Y Y = 3	6 ÷ X = 2 X = 3 (Because 6 ÷ 2 = 3.)	8 ÷ X = Z X = 2 Z = 4 (Because 8 ÷ 2 = 8.)	X ÷ Z = Z Z = 4 X = 16 (Because 16 ÷ 4 = 4.)
1. 10 ÷ 5 = Y Y = ___	2. 4 ÷ 4 = X X = ___	3. 6 ÷ 1 = Z Z = ___	4. 12 ÷ Y = 6 Y = ___
5. Z ÷ 2 = 5 Z = ___	6. 8 ÷ X = 8 X = ___	7. 25 ÷ Y = Y Y = ___	8. X ÷ 1 = X X = ___
9. X ÷ 4 = Z X = 16 Z = ___	10. X ÷ Y = 2 X = 8 Y = ___	11. X ÷ Z = 5 X = ___ Z = 3	12. Z ÷ Y = Z Z = 3 Y = ___
13. X ÷ X = Z X = 5 Z = ___	14. Y ÷ Y = 1 ___Y__ = ___	15 Z_÷ Z_= Y ___Z__ = ___ ___Y__ = ___	16. Y_÷ Y_= Z ___Z__ = ___

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